

Query Answering in Resource-Based Answer Set Semantics *

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Abstract

In recent work we defined resource-based answer set semantics, which is an extension to answer set semantics stemming from the study of its relationship with linear logic. In fact, the name of the new semantics comes from the fact that in the linear-logic formulation every literal (including negative ones) were considered as a resource. In this paper, we propose a query-answering procedure reminiscent of Prolog for answer set programs under this extended semantics as an extension of XSB-resolution for logic programs with negation.¹ We prove formal properties of the proposed procedure.

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KEYWORDS: Answer Set Programming, Procedural Semantics, Top-down Query-answering

1 Introduction

Answer set programming (ASP) is nowadays a well-established and successful programming paradigm based on answer set semantics (Gelfond and Lifschitz 1988; Marek and Truszczyński 1999), with applications in many areas (cf., e.g., (Baral 2003; Truszczyński 2007; Gelfond 2007) and the references therein). Nevertheless, as noted in (Gebser et al. 2009; Bonatti et al. 2008), few attempts to construct a goal-oriented proof procedure exist, though there is a renewal of interest, as attested, e.g., by the recent work presented in (Marple and Gupta 2014). This is due to the very nature of the answer set semantics, where a program may admit none or several answer sets, and where the semantics enjoys no locality, or, better, no *Relevance* in the sense of (Dix 1995): no subset of the given program can in general be identified, from where the decision of atom A (intended as a goal, or query) belonging or not to some answer set can be drawn. An incremental construction of approximations of answer sets is proposed in (Gebser et al. 2009) to provide a ground for local computations and top-down query answering. A sound and complete proof procedure is also provided. The approach of (Bonatti et al. 2008) is in the spirit of “traditional” SLD-resolution (Lloyd 1993),

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and can be used with non-ground queries and with non-ground, possibly infinite, programs. Soundness and completeness results are proven for large classes of programs. Another way to address the query-answering problem is discussed in (Lin and You 2002). This work describes a canonical rewriting system that turns out to be sound and complete under the *partial stable model semantics*. In principle, as the authors observe, the inference procedure could be completed to implement query-answering w.r.t. stable model semantics by circumventing the lack of Relevance. A substantially different approach to ASP computation is proposed in (Gebser and Schaub 2006) where the authors define a tableau-based framework for ASP. The main aim consists in providing a formal framework for characterizing inference operations and strategies in ASP-solvers. The approach is not based on query-oriented top-down evaluation, indeed, each branch in a tableau potentially corresponds to a computation of an answer set. However, one might foresee the possibility of exploiting such a tableau system to check answer set existence subject to query satisfaction.

A relevant issue concerning goal-oriented answer-set-based computation is related to sequences of queries. Assume that one would be able to pose a query $?-Q_1$ receiving an answer “yes”, to signify that Q_1 is entailed by some answer set of the given program Π . Possibly, one might intend subsequent queries to be answered in the same *context*, i.e. a subsequent query $?-Q_2$ might ask whether some of the answer sets entailing Q_1 also entails Q_2 . This might go on until the user explicitly “resets” the context. Such an issue, though reasonable in practical applications, has been hardly addressed up to now, due to the semantic difficulties that we have mentioned. A viable approach to these problems takes inspiration from the research on RASP (Resource-based ASP), which is a recent extension of ASP, obtained by explicitly introducing the notion of *resource* (Costantini and Formisano 2010). A RASP and linear-logic modeling of default negation as understood under the answer set semantics has been introduced in (Costantini and Formisano 2013). This led to the definition of an extension to the answer set semantics, called *Resource-based Answer Set Semantics* (RAS). The name of the new semantics comes from the fact that in the linear-logic formulation every literal (including negative ones) is considered as a resource that is “consumed” (and hence it becomes no more available) once used in a proof. This extension finds an alternative equivalent definition in a variation of the auto-epistemic logic characterization of answer set semantics discussed in (Marek and Truszczyński 1993).

We refer the reader to (Costantini and Formisano 2015) for a discussion of the new semantics from several points of view, and to the Appendix for a summary of its formal definition. Under resource-based answer set semantics there are no inconsistent programs, i.e., every program admits (resource-based) answer set. Consider for instance the program $\Pi_1 = \{old \leftarrow notold\}$. Under the answer set semantics, this program is inconsistent (has no answer sets) because it consists of a unique odd cycle and no supported models exists. If we extend the program to $\Pi_2 = \{old \leftarrow notold, old \leftarrow notyoung\}$ then the resulting program has the answer set $\{old\}$: in fact, the first rule is overridden by the second rule which allows *old* to be derived. Under the resource-based answer set semantics the first rule is ignored in the first place: in fact, Π_1 has a unique resource-based answer set which is the empty set. Intuitively, this results from interpreting default negation *notA* as “I assume that *A* is false” or, in autoepistemic terms (Marek and Truszczyński 1991a; Marek and Truszczyński 1991b) “I believe that I don’t believe *A*”. So, since deriving *A* accounts to denying the assumption of *notA*, such a derivation is disallowed as it would be contradictory. It is not considered to be inconsistent because default negation is not negation in classical logic: in fact, the attempt of deriving *A* from *notA* in classical

logic leads to an inconsistency, while contradicting one's own assumption is (in our view) simply meaningless, so a rule such as the one in Π_1 is plainly ignored. Assume now to further enlarge the program, by obtaining $\Pi_3 = \{old \leftarrow not\ old. \ old \leftarrow not\ young. \ young \leftarrow old.\}$. There are again no answer sets, because by combining the last two rules a contradiction on *young* is determined, though indirectly. In resource-based answer set semantics there is still the answer set $\{old\}$, as the indirect contradiction is ignored: having assumed *not young* makes *young* unprovable.

In standard ASP, a constraint such as $\leftarrow L_1, \dots, L_h$ where the L_i s are literals is implemented by translating it into the rule $p \leftarrow not\ p, L_1, \dots, L_h$ with p fresh atom. This is because, in order to make the contradiction on p harmless, one of the L_i s must be false: otherwise, no answer set exists. Under resource-based answer set semantics such a transposition no longer works. Thus, constraints related to a given program are not seen as part of the program: rather, they must be defined separately and associated to the program. Since resource-based answer sets always exist, constraints will possibly exclude (a-posteriori) some of them. Thus, constraints act as a filter on resource-based answer sets, leaving those which are *admissible* with respect to given constraints.

In this paper we discuss a top-down proof procedure for the new semantics. The proposed procedure, beyond query-answering, also provides contextualization, via a form of tabling; i.e., a table is associated with the given program, and initialized prior to posing queries. Such table contains information useful for both the next and the subsequent queries. Under this procedure, $?-A$ (where we assume with no loss of generality that A is an atom), succeeds whenever there exists some resource-based answer set M where $A \in M$. Contextualization implies that given a sequence of queries, for instance $?-A, ?-B$, both queries succeed if there exists some resource-based answer set M where $A \in M \wedge B \in M$: this at the condition of evaluating $?-B$ on the program table as left by $?-A$ (analogously for longer sequences). In case the table is reset, subsequent queries will be evaluated independently of previous ones. Success of $?-A$ must then be validated with respect to constraints; this issue is only introduced here, and will be treated in a future paper.

Differently from (Gebser et al. 2009), the proposed procedure does not require incremental answer set construction when answering a query and is not based on preliminary program analysis as done in (Marple and Gupta 2014). Rather, it exploits the fact that resource-based answer set semantics enjoys the property of Relevance (Dix 1995) (whereas answer set semantics does not). This guarantees that the truth value of an atom can be established on the basis of the subprogram it depends upon, and thus allows for top-down computation starting from a query. For previous sample programs Π_2 and Π_3 , query $?-old$ succeeds, while $?-young$ fails. W.r.t. the top-down procedure proposed in (Bonatti et al. 2008), we do not aim at managing function symbols (and thus programs with infinite grounding), so concerning this aspect our work is more limited.

As answer set semantics and resource-based answer set semantics extend the well-founded semantics (Van Gelder et al. 1991), we take as a starting point XSB-resolution (Swift and Warren 2012; Chen and Warren 1993), an efficient, fully described and implemented procedure which is correct and complete w.r.t. the well-founded semantics. In particular, we define RAS-XSB-resolution and discuss its properties; we prove correctness and completeness for every program (under the new semantics). We do not provide the full implementation details that we defer to a next step; in fact, this would imply suitably extending and reworking all operative aspects related to XSB. Thus, practical issues such as efficiency and optimization are not dealt with in the present paper and are rather deferred to future work of actual specification of an implementation. The proposed procedure is intended as a proof-of-concept rather than as an implementation guideline.

RAS-XSB resolution can be used for answer set programming under the software engineering discipline of dividing the program into a consistent “base” level and a “top” level including constraints. Therefore, even to readers not particularly interested in the new semantics, the paper proposes a full top-down query-answering procedure for ASP, though applicable under such (reasonable) limitation.

In summary, RAS-XSB-Resolution:

- can be used for (credulous) top-down query-answering on logic programs under the resource-based answer set semantics and possibly under the answer set semantics, given the condition that constraints are defined separately from the “main” program;
- it is meant for the so-called “credulous reasoning” in the sense that given, say, query $?-A$ (where A is an atom), it determines whether there exists any (resource-based) answer set M such that $A \in M$;
- it provides “contextual” query-answering, i.e. it is possible to pose subsequent queries, say $?-A_1, \dots, ?-A_n$ and, if they all succeed, this means that there exists some (resource-based) answer set M such that $\{A_1, \dots, A_n\} \subseteq M$; this extends to the case when only some of them succeed, where successful atoms are all in M and unsuccessful ones are not;
- does not require either preliminary program analysis or incremental answer-set construction, and does not impose any kind of limitation over the class of resource-based answer set programs which are considered (for answer set programs, there is the above-mentioned limitation on constraints).

This paper is organized as follows. After a presentation of resource-based answer set semantics in Section 2, we present the proposed query-answering procedure in Section 3, and conclude in Section 4. In the rest of the paper, we refer to the standard definitions concerning propositional general logic programs and ASP (Lloyd 1993; Apt and Bol 1994; Gelfond 2007). If not differently specified, we will implicitly refer to the *ground* version of a program Π . We do not consider “classical negation”, double negation *notnot* A , disjunctive programs, or the various useful programming constructs, such as aggregates, added over time to the basic ASP paradigm (Simons et al. 2002; Costantini and Formisano 2011; Faber et al. 2011).

2 Background on Resource-based ASP

The denomination “resource-based” answer set semantics (RAS) stems from the linear logic formulation of ASP (proposed in (Costantini and Formisano 2013; Costantini and Formisano 2015)), which constituted the original inspiration for the new semantics. In this perspective, the negation *not* A of some atom A is considered to be a *resource* of unary amount, where:

- *not* A is *consumed* whenever it is used in a proof, thus preventing A to be proved, for retaining consistency;
- *not* A becomes no longer available whenever A is proved.

Consider for instance the following well-known sample answer set program consisting of a ternary odd cycle and concerning someone who wonders where to spend her vacation:

beach \leftarrow *not**mountain*. *mountain* \leftarrow *not**travel*. *travel* \leftarrow *not**beach*.

In ASP, such program is inconsistent. Under the new semantics, there are the following three resource-based answer sets: $\{beach\}$, $\{mountain\}$, and $\{travel\}$. Take for instance the first one,

$\{beach\}$. In order to derive the conclusion *beach* the first rule can be used; in doing so, the premise *notmountain* is consumed, thus disabling the possibility of proving *mountain*, which thus becomes false; *travel* is false as well, since it depends from a false premise.

We refer the reader to (Costantini and Formisano 2015) for a detailed discussion about logical foundations, motivations, properties, and complexity, and for examples of use. We provide therein characterizations of RAS in terms of linear logic, as a variation of the answer set semantics, and in terms of autoepistemic logic. Here we just recall that, due to the ability to cope with odd cycles, under RAS it is always possible to assign a truth value to all atoms: every program in fact admits at least one (possibly empty) resource-based answer set. A more significant example is the following (where, albeit in this paper we focus on the case of ground programs, for the sake of conciseness we make use of variables, as customary done to denote collections of ground literals/rules). The program models a recommender agent, which provides a user with indication to where it is possible to spend the evening, and how the user should dress for such an occasion. The system is also able to take user preferences into account.

The resource-based answer set program which constitutes the core of the system is the following. There are two ternary cycles. The first one specifies that a person can be dressed either formally or normally or in an eccentric way. In addition, only old-fashioned persons dress formally, and only persons with a young mind dress in an eccentric way. Later on, it is stated by two even cycles that any person can be old-fashioned or young-minded, independently of the age that, by the second odd cycle, can be young, middle, or old. The two even cycles interact, so that only one option can be taken. Then, it is stated that one is admitted to an elegant restaurant if (s)he is formally dressed, and to a disco if (s)he is dressed in an eccentric way. To spend the evening either in an elegant restaurant or in a disco one must be admitted. Going out in this context means either going to an elegant restaurant (for middle-aged or old people) or to the disco for young people, or sightseeing for anyone.

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formal_dress(P) ← person(P), notnormal_dress(P), old_fashioned(P).
normal_dress(P) ← person(P), noteccentric_dress(P).
eccentric_dress(P) ← person(P), notformal_dress(P), young_mind(P).
old(P) ← person(P), notmiddleaged(P).
middleaged(P) ← person(P), notyoung(P).
young(P) ← person(P), notold(P).
old_fashioned(P) ← person(P), notyoung_mind(P), notnoof(P).
noof(P) ← person(P), notold_fashioned(P).
young_mind(P) ← person(P), notold_fashioned(P), notnoym(P).
noym(P) ← person(P), notyoung_mind(P).
admitted_elegant_restaurant(P) ← person(P), formal_dress(P).
admitted_disco(P) ← person(P), eccentric_dress(P).
go_disco(P) ← person(P), young(P), admitted_disco(P).
go_elegant_restaurant(P) ← person(P), admitted_elegant_restaurant(P).
go_elegant_restaurant(P) ← person(P), middleaged(P), admitted_elegant_restaurant(P).
go_sightseeing(P) ← person(P).
go_out(P) ← middleaged(P), go_elegant_restaurant(P).
go_out(P) ← old(P), go_elegant_restaurant(P).
go_out(P) ← young(P), go_disco(P).
go_out(P) ← go_sightseeing(P).

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The above program, if considered as an answer set program, has a (unique) empty resource-based answer set, as there are no facts (in particular there are no facts for the predicate *person* to provide values for the placeholder *P*).

Now assume that the above program is incorporated into an interface system which interacts with a user, say George, who wants to go out and wishes to be made aware of his options. The system may thus add the fact *person(george)* to the program. While, in ASP the program would become inconsistent, in RASP the system would, without any more information, advise George to go sightseeing. This is, in fact, the only advice that can be extracted from the unique resource-based answer set of the resulting program. If the system might obtain or elicit George's age, the options would be many more, according to the hypotheses about him being old-fashioned or young-minded. Moreover, for each option (except sightseeing) the system would be able to extract the required dress code. George might want to express a preference, e.g., going to the disco. Then the system might add to the program the rule

$$preference(P) \leftarrow person(P), go_disco(P).$$

and state the constraint $\leftarrow notpreference(P)$ that “forces” the preference to be satisfied, thus making George aware of the hypotheses and conditions under which he might actually go to the disco. Namely, they correspond to the unique resource-based answer set where George is young, young-minded and dresses in an eccentric way.

However, in resource-based answer set semantics constraints cannot be modeled (as done in ASP) as “syntactic sugar”, in terms of unary odd cycles involving fresh atoms. Hence, they have to be modeled explicitly. Without loss of generality, we assume, from now on, the following simplification concerning constraints. Each constraint $\leftarrow L_1, \dots, L_k$, where each L_i is a literal, can be rephrased as simple constraint $\leftarrow H$, where H is a fresh atom, plus rule $H \leftarrow L_1, \dots, L_k$ to be added to the given program Π . So, H occurs in the set S_Π of all the atoms of Π .

Definition 2.1

Let Π be a program and $\mathcal{C} = \{\mathcal{C}_1, \dots, \mathcal{C}_k\}$ be a set of constraints, each \mathcal{C}_i in the form $\leftarrow H_i$.

- A resource-based answer set M for Π is *admissible* w.r.t. \mathcal{C} if for all $i \leq k$ where $H_i \notin M$.
- The program Π is called “admissible” w.r.t. \mathcal{C} if it has an admissible answer w.r.t. \mathcal{C} .

It is useful for what follows to evaluate RAS with respect to general properties of semantics of logic programs introduced in (Dix 1995), that we recall below (see the mentioned article for the details). A semantic *SEM* for logic programs is intended as a function which associates a logic program with a set of sets of atoms, which constitute the intended meaning.

Definition 2.2

Given any semantics *SEM* and a ground program Π , *Relevance* states that for all literals L it holds that $SEM(\Pi)(L) = SEM(rel_rul(\Pi; L))(L)$.

Relevance implies that the truth value of any literal under that semantics in a given program, is determined solely by the subprogram consisting of the relevant rules. The answer set semantics does not enjoy Relevance (Dix 1995). This is one reason for the lack of goal-oriented proof procedures. Instead, it is easy to see that resource-based answer set semantics enjoys Relevance.

Resource-based answer set semantics, like most semantics for logic programs with negation, enjoys *Reduction*, which simply assures that the atoms not occurring in the heads of a program are always assigned truth value false.

Another important property is *Modularity*, defined in (Dix 1995) as follows (where the reduct Π^M of program Π w.r.t. set of atoms M):

Definition 2.3

Given any semantics SEM , a ground program Π let $\Pi = \Pi_1 \cup \Pi_2$ where for every atom A occurring in Π_2 , $rel_rul(\Pi; A) \subseteq \Pi_2$. We say that SEM enjoys *Modularity* if it holds that $SEM(\Pi) = SEM(\Pi_1^{SEM(\Pi_2)} \cup \Pi_2)$.

If Modularity holds, then the semantics can be always computed by splitting a program in its sub-programs (w.r.t. relevant rules). Intuitively, in the above definition, the semantics of Π_2 , which is self-contained, is first computed. Then, the semantics of the whole program can be determined by reducing Π_1 w.r.t. $SEM(\Pi_2)$. We can state (as a consequence of Relevance and of Proposition A.3 in the Appendix) that resource-based answer set semantics enjoys Modularity.

Proposition 2.1

Given a ground program Π let $\Pi = \Pi_1 \cup \Pi_2$, where for every atom A occurring in Π_2 , $rel_rul(\Pi; A) \subseteq \Pi_2$. A set M of atoms is a resource-based answer set of Π iff there exists a resource-based answer set S of Π_2 such that M is a resource-based answer set of $\Pi_1^S \cup \Pi_2$.

Modularity also impacts on constraint checking, i.e. on the check of admissibility of resource-based answer sets. Considering, in fact, a set of constraints $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$, $n > 0$, each \mathcal{C}_i in the form $\leftarrow H_i$, and letting for each $i \leq n$ $rel_rul(\Pi; H_i) \subseteq \Pi_2$, from Proposition 2.1 it follows that, if a resource-based answer set X of Π_2 is admissible (in terms of Definition 2.1) w.r.t. $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$, then any resource-based answer set M of Π such that $X \subseteq M$ is also admissible w.r.t. $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$. In particular, Π_2 can be identified in relation to a certain query:

Definition 2.4

Given a program Π , a constraint $\leftarrow H$ associated to Π is *relevant for query* $?-A$ if $rel_rul(\Pi; A) \subseteq rel_rul(\Pi; H)$.

3 A Top-down Proof Procedure for RAS

As it is well-known, the answer set semantics extends the well-founded semantics (wfs) (Van Gelder et al. 1991) that provides a unique three-valued model $\langle W^+, W^- \rangle$, where atoms in W^+ are *true*, those in W^- are *false*, and all the others are *undefined*. In fact, the answer set semantics assigns, for consistent programs truth values to the undefined atoms. However the program can be *inconsistent* because of odd cyclic dependencies. The improvement of resource-based answer set semantics over the answer set semantics relies exactly on its ability to deal with odd cycles that the answer set semantics interprets as inconsistencies. So, in any reasonable potential query-answering device for ASP, a query $?-A$ to an ASP program Π may be reasonably expected to succeed or fail if A belongs to W^+ or W^- , respectively. Such a procedure will then be characterized according to how to provide an answer when A is undefined under the wfs.

An additional problem with answer set semantics is that query $?-A$ might *locally* succeed, but still, for the lack of Relevance, the overall program may not have answer sets. In resource-based answer set semantics instead, every program has one or more resource-based answer set: each of them taken singularly is then admissible or not w.r.t. the integrity constraints. This allows one to perform constraint checking upon success of query $?-A$.

We will now define the foundations of a top-down proof procedure for resource-based answer set semantics, which we call RAS-XSB-resolution. The procedure has to deal with atoms involved in negative circularities, that must be assigned a truth value according to some resource-based answer set. We build upon XSB-resolution, for which an ample literature exists, from the

seminal work in (Chen and Warren 1993) to the most recent work in (Swift and Warren 2012) where many useful references can also be found. For lack of space XSB-resolution is not described here. XSB in its basic version, XOLDTNF-resolution (Chen and Warren 1993) is shortly described in the Appendix. We take for granted basic notions concerning proof procedures for logic programming, such as for instance backtracking. For the relevant definitions we refer to (Lloyd 1993). Some notions are however required here for the understanding of what follows. In particular, it is necessary to illustrate detection of cycles on negation.

Definition 3.1 (XSB Negative Cycles Detection)

- Each call to atom A has an associated set N of negative literals, called the *negative context* for A , so the call takes the form (A, N) .
- Whenever a negative literal $notB$ is selected during the evaluation of some A , there are two possibilities: (i) $notB \notin N$: this will lead to the call $(B, N \cup \{notB\})$; (ii) $notB \in N$, then there is a possible negative loop, and B is called a *possibly looping negative literal*.
- For the initial call of any atom A , N is set to empty.

In order to assume that a literal $notB$ is a looping negative literal, that in XSB assumes truth value *undefined*, the evaluation of B must however be *completed*, i.e. the search space must have been fully explored without finding conditions for success or failure.

Like in XSB, for each program Π a table $\mathcal{T}(\Pi)$ records useful information about proofs. As a small extension w.r.t. XSB-Resolution, we record in $\mathcal{T}(\Pi)$ not only successes, but also failures. XSB-resolution is, for Datalog programs, correct and complete w.r.t. the wfs. Thus, it is useful to state the following definition.

Definition 3.2

Given a program Π and an atom A , we say that

- A *definitely succeeds* iff it succeeds via XSB- (or, equivalently, XOLDTNF-) resolution, and thus A is recorded in $\mathcal{T}(\Pi)$ with truth value *true*. For simplicity, we assume A occurs in $\mathcal{T}(\Pi)$.
- A *definitely fails* iff it fails via XSB- (or, equivalently, XOLDTNF-) resolution, and thus A is recorded in $\mathcal{T}(\Pi)$ with truth value *false*. For simplicity, we assume $notA$ occurs in $\mathcal{T}(\Pi)$.

To represent the notion of negation as a resource, we initialize the program table prior to posing queries and we manage the table during a proof so as to state that:

- the negation of any atom which is not a fact is available unless this atom has been proved;
- the negation of an atom which has been proved becomes unavailable;
- the negation of an atom which cannot be proved is always available.

Definition 3.3 (Table Initialization in RAS-XSB-resolution)

Given a program Π and an associated table $\mathcal{T}(\Pi)$, *Initialization* of $\mathcal{T}(\Pi)$ is performed by inserting, for each atom A occurring as the conclusion of some rule in Π , a fact $yesA$ (where $yesA$ is a fresh atom).

The meaning of $yesA$ is that the negation $notA$ of A has not been proved. If $yesA$ is present in the table, then A can possibly succeed. Success of A “absorbs” $yesA$ and prevents $notA$ from success. Failure of A or success of $notA$ “absorbs” $yesA$ as well, but $notA$ is asserted. $\mathcal{T}(\Pi)$ will in fact evolve during a proof into subsequent states, as specified below.

Definition 3.4 (Table Update in RAS-XSB-resolution)

Given a program Π and an associated table $\mathcal{T}(\Pi)$, referring to the definition of RAS-XSB-resolution (cf. Definition 3.5 below), the table update is performed as follows.

- Upon success of subgoal A , $yesA$ is removed from $\mathcal{T}(\Pi)$ and A is added to $\mathcal{T}(\Pi)$.
- Upon failure of subgoal A , $yesA$ is removed from $\mathcal{T}(\Pi)$ and $notA$ is added to $\mathcal{T}(\Pi)$.
- Upon success of subgoal $notA$, $yesA$ is removed from $\mathcal{T}(\Pi)$ and $notA$ is added to $\mathcal{T}(\Pi)$.

However:

- if $notA$ succeeds by case 3.b, then such modification is permanent;
- if $notA$ succeeds either by case 3.c or by case 3.d, then in case of failure of the parent subgoal the modification is retracted, i.e. $yesA$ is restored in $\mathcal{T}(\Pi)$ and $notA$ is removed from $\mathcal{T}(\Pi)$.

We refer the reader to the examples provided below for a clarification of the table-update mechanism. In the following, without loss of generality we can assume that a query is of the form $?-A$, where A is an atom. Success or failure of this query is established as follows. Like in XSB-resolution, we assume that the call to query A implicitly corresponds to the call (A, N) where N is the negative context of A , which is initialized to \emptyset and treated as stated in Definition 3.1.

Definition 3.5 (Success and failure in RAS-XSB-resolution)

Given a program Π and its associated table $\mathcal{T}(\Pi)$, notions of success and failure and of modifications to $\mathcal{T}(\Pi)$ are extended as follows with respect to XSB-resolution.

- (1) Atom A succeeds iff $yesA$ is present in $\mathcal{T}(\Pi)$, and one of the following conditions holds.
 - (a) A definitely succeeds (which includes the case where A is present in $\mathcal{T}(\Pi)$).
 - (b) There exists in Π either fact A or a rule of the form $A \leftarrow L_1, \dots, L_n, n > 0$, such that neither A nor $notA$ occur in the body and every literal $L_i, i \leq n$, succeeds.
- (2) Atom A fails iff one of the following conditions holds.
 - (a) $yesA$ is not present in $\mathcal{T}(\Pi)$.
 - (b) A definitely fails.
 - (c) There is no rule of the form $A \leftarrow L_1, \dots, L_n, n > 0$, such that every literal L_i succeeds.
- (3) Literal $notA$ succeeds if one of the following is the case:
 - (a) $notA$ is present in $\mathcal{T}(\Pi)$.
 - (b) A fails.
 - (c) $notA$ is *allowed to succeed*.
 - (d) A is *forced to failure*.
- (4) Literal $notA$ fails if A succeeds.
- (5) $notA$ is *allowed to succeed* whenever the call (A, \emptyset) results, whatever sequence of derivation steps is attempted, in the call $(A, N \cup \{notA\})$. I.e., the derivation of $notA$ incurs through layers of negation again into $notA$.
- (6) A is *forced to failure* when the call (A, \emptyset) always results in the call $(A, \{notA\})$, whatever sequence of derivation steps is attempted. I.e., the derivation of $notA$ incurs in $notA$ directly.

From the above extension of the notions of success and failure we obtain RAS-XSB-resolution as an extended XSB-resolution. Actually, in the definition we exploit XSB (or, more precisely, XOLDTNF), as a “plugin” for definite success and failure, and we add cases which manage subgoals with answer *undefined* under XSB. This is not exactly ideal from an implementation point of view. In future work, we intend to proceed to a much more effective integration of XSB with the new aspects that we have introduced, and to consider efficiency and optimization issues that are presently neglected.

Notice that the distinction between RAS-XSB-resolution and XSB-resolution is determined by cases 3.c and 3.d of Definition 3.5, which manage literals involved in negative cycles. The notions of *allowance to succeed* (case 5) and of *forcing to failure* (case 6) are crucial. Let us illustrate the various cases via simple examples:

- Case 3.c deals with literals depending negatively upon themselves through other negations. Such literals can be assumed as *hypotheses*. Consider, for example, the program $a \leftarrow notb. \quad b \leftarrow nota.$ Query $?-a$ succeeds by assuming *notb*, which is correct w.r.t. (resource-based) answer set $\{a\}$. If, however, the program is $a \leftarrow notb, note. \quad b \leftarrow nota. \quad e.$ then, the same query $?-a$ fails upon definite failure of *note*, so the hypothesis *notb* must be retracted. This is, in fact, stated in the specification of table update (Definition 3.4).
- Case 3.d deals with literals depending negatively upon themselves directly. Such literals can be assumed as *hypotheses*. Consider, for example, the program $p \leftarrow a. \quad a \leftarrow notp.$ Query $?-a$ succeeds because the attempt to prove *notp* comes across *notp* (through *a*), and thus *p* is forced to failure. This is correct w.r.t. resource-based answer set $\{a\}$. Notice that for atoms involved in negative cycles the positive-cycle detection is relaxed, as some atom in the cycle will either fail or been forced to failure. If however the program is $p \leftarrow a. \quad a \leftarrow notp, notq.$ then, the same query $?-a$ fails upon definite failure of *notq*, so the hypothesis *notp* must be retracted. This is in fact stated in the specification of table update (Definition 3.4).

We provide below a high-level definition of the overall proof procedure (overlooking implementation details), which resembles plain SLD-resolution.

Definition 3.6 (A naive RAS-XSB-resolution)

Given a program Π , let assume as input the data structure $\mathcal{T}(\Pi)$ used by the proof procedure for tabling purposes, i.e. the table associated with the program. Given a query $?-A$, the list of current subgoals is initially set to $\mathcal{L}_1 = \{A\}$. If in the construction of a proof-tree for $?-A$ a literal L_{ij} is selected in the list of current subgoals \mathcal{L}_i , we have that: if L_{ij} succeeds then we take L_{ij} as proved and proceed to prove L_{i+1} after the related updates to the program table. Otherwise, we have to backtrack to the previous list \mathcal{L}_{i-1} of subgoals.

Conditions for success and failure are those specified in Definition 3.5. Success and failure determine the modifications to $\mathcal{T}(\Pi)$ specified in Definition 3.4. Backtracking does not involve restoring previous contents of $\mathcal{T}(\Pi)$, as subgoals which have been proved can be usefully employed as lemmas. In fact, the table is updated only when the entire search space for a subgoal has been explored. The only exception concerns negative subgoals which correspond to literals involved in cycles: in fact, they are to be considered as hypotheses that could later be retracted.

For instance, consider the program

$$q \leftarrow nota, c. \quad q \leftarrow notb. \quad a \leftarrow notb. \quad b \leftarrow nota.$$

and query $?-q$. Let us assume clauses are selected in the order. So, the first clause for *q* is

selected, and *nota* is initially allowed to succeed (though involved in a negative cycle with *notb*). However, upon failure of subgoal *c* with consequent backtracking to the second rule for *q*, lemma *notA* must be retracted from the table: this in fact enables *notb* to be allowed to succeed, so determining success of the query.

Definition 3.7

Given a program Π and its associated table $\mathcal{T}(\Pi)$, a *free query* is a query $?-A$ which is posed on Π when the table has just been initialized. A *contextual query* is a query $?-B$ which is posed on Π leaving the associated table in the state determined by former queries.

Success of query $?-A$ means (as proved in Theorem 3.1 below) that there exist resource-based answer sets that contain *A*. The final content of $\mathcal{T}(\Pi)$ specifies literals that hold in these sets (including *A*). Precisely, the state of $\mathcal{T}(\Pi)$ characterizes a set $\mathcal{S}_{\mathcal{T}(\Pi)_A}$ resource-based answer sets of Π , such that for all $M \in \mathcal{S}_{\mathcal{T}(\Pi)_A}$, and for every atom *D*, $D \in \mathcal{T}(\Pi)$ implies $D \in M$ and $\text{not}D \in \mathcal{T}(\Pi)$ implies $D \notin M$. Backtracking on $?-A$ accounts to asking whether there are other different resource-based answer sets containing *A*, and implies making different assumptions about cycles by retracting literals which had been assumed to succeed. Instead, posing a subsequent query $?-B$ without resetting the contents of $\mathcal{T}(\Pi)$, which constitutes a *context*, accounts to asking whether some of the answer sets in $\mathcal{S}_{\mathcal{T}(\Pi)_A}$ also contain *B*. Posing such a contextual query, the resulting table reduces previously-identified resource-based answer sets to a possibly smaller set $\mathcal{S}_{\mathcal{T}(\Pi)_{A \cup B}}$ whose elements include both *A* and *B* (see Theorem 3.2 below). Contextual queries and sequences of contextual queries are formally defined below.

Definition 3.8 (Query sequence)

Given a program Π and $k > 1$ queries $?-A_1, \dots, ?-A_k$ performed one after the other, assume that $\mathcal{T}(\Pi)$ is initialized only before posing $?-A_1$. Then, $?-A_1$ is a free query where each $?-A_i$, is a *contextual query*, evaluated w.r.t. the previous ones.

To show the application of RAS-XSB-resolution to single queries and to a query sequence, let us consider the sample following program Π , which includes virtually all cases of potential success and failure. The well-founded model of this program is $\langle\{e\}, \{d\}\rangle$ while the resource-based answer sets are $M_1 = \{a, e, f, h, s\}$ and $M_2 = \{e, h, g, s\}$.

$r_1. \ a \leftarrow \text{not}g.$	$r_3. \ s \leftarrow \text{not}p.$	$r_5. \ h \leftarrow \text{not}p.$	$r_7. \ f \leftarrow \text{not}g, e.$
$r_2. \ g \leftarrow \text{not}a.$	$r_4. \ p \leftarrow h.$	$r_6. \ f \leftarrow \text{not}a, d.$	$r_8. \ e.$

Initially, $\mathcal{T}(\Pi)$ includes *yesA* for every atom occurring in some rule head: $\mathcal{T}(\Pi) = \{\text{yes}a, \text{yes}b, \text{yes}c, \text{yes}e, \text{yes}f, \text{yes}g, \text{yes}p, \text{yes}h, \text{yes}s\}$. Below we illustrate some derivations. We assume that applicable rules are considered from first (r_1) to last (r_8) as they are ordered in the program, and literals in rule bodies from left to right.

Let us first illustrate the proof of query $?-f$. Each additional layer of $?-$ indicates nested derivation of *A* whenever literal *notA* is encountered. In the comment, we refer to cases of RAS-XSB-resolution as specified in Definition 3.5. Let us first consider query $?-f$.

```
?- f.
?- nota, d.      % via r6
Subgoal nota is treated as follows.
?- ?- a.
?- ?- notg.      % via r1
?- ?- ?- g.
?- ?- ?- nota.   % via r2. nota succeeds by case 3.c,  $\mathcal{T}(\Pi) = \mathcal{T}(\Pi) \cup \{\text{not}a\} \setminus \{\text{yes}a\}$ 
```

Subgoal d gives now rise to the following derivation.

$?-d.$ % d fails by case 2.b, so the parent goal f fails.

Backtracking is however possible, as there exists a second rule for f .

$?-notg, e.$ % via r_7

$?-?-g.$

$?-?-nota.$ % via r_2

$?-?-?-a.$

$?-?-?-notg.$ % via r_1 . Thus, $notg$ succeeds by case 3.c. $\mathcal{T}(\Pi) = \mathcal{T}(\Pi) \cup \{notg\} \setminus \{yesg\}$

Now, the second subgoal e remains to be completed:

$?-e.$ % e succeeds by case 1.b, and the overall query f succeeds by case 1.b.

$$\mathcal{T}(\Pi) = \mathcal{T}(\Pi) \cup \{e, f\} \setminus \{yese, yesf\}$$

Assuming now to go on to query the same context, i.e. without re-initializing $\mathcal{T}(\Pi)$, query $?-g$ quickly fails by case 2.a since $notg \in \mathcal{T}(\Pi)$. Query $?-e$ succeeds immediately by case 1.a as $e \in \mathcal{T}(\Pi)$. We can see that the context we are within corresponds to resource-based answer set M_1 . Notice that, if resetting the context, $?-g$ would instead succeed as by case 1.b as $nota$ can be allowed to succeed by case 3.c. Finally, a derivation for $?-s$ is obtained as follows:

$?-s.$

$?-notp.$ % via r_3

$?-?-p.$

$?-?-h.$ % via r_4

$?-?-notp.$ % via r_5 , $notp$ succeeds by case 3.d, and p is forced to failure

$$\mathcal{T}(\Pi) = \mathcal{T}(\Pi) \cup \{notp\} \setminus \{yesp, yesp\}.$$

Then, at the upper level, s and h succeed by case 1.b, and $\mathcal{T}(\Pi) \cup \{s\} \setminus \{yess\}$. Notice that forcing p to failure determines $notp$ to succeed, and consequently allows h to succeed (where h is undefined under the wfs). The derivation of h involves the tricky case of a positive dependency through negation.

3.1 Properties of RAS-XSB-resolution

Properties of resource-based answer set semantics are strictly related to properties of RAS-XSB-resolution. In fact, thanks to Relevance we have soundness and completeness, and Modularity allows for contextual query and locality in constraint-checking. Such properties are summarized in the following Theorems (whose proofs can be found in Appendix).

Theorem 3.1

RAS-XSB-resolution is correct and complete w.r.t. resource-based answer set semantics, in the sense that, given a program Π , a query $?-A$ succeeds under RAS-XSB-resolution with an initialized $\mathcal{T}(\Pi)$ iff there exists resource-based answer set M for Π where $A \in M$.

Theorem 3.2

RAS-XSB-resolution is contextually correct and complete w.r.t. resource Answer Set semantics, in the sense that, given a program Π and a query sequence $?-A_1, \dots, ?-A_k$, $k > 1$, where $\{A_1, \dots, A_k\} \subseteq S_\Pi$ (i.e. the A_i s are atoms occurring in Π), we have that, for $\{B_1, \dots, B_r\} \subseteq \{A_1, \dots, A_k\}$ and $\{D_1, \dots, D_s\} \subseteq \{A_1, \dots, A_k\}$, the queries $?-B_1, \dots, ?-B_r$ succeed while $?-D_1, \dots, ?-D_s$ fail under RAS-XSB-resolution, iff there exists resource-based answer set M for Π where $\{B_1, \dots, B_r\} \subseteq M$ and $\{D_1, \dots, D_s\} \cap M = \emptyset$.

This result extends immediately to queries including negative literals such as $not H$, $H \in S_\Pi$. We say that a query sequence contextually succeeds if each of the involved queries succeeds in the context (table) left by all former ones.

We defer a discussion of constraint checking to a future paper. Notice only that, given an admissible program Π and a constraint $\leftarrow C$ (where C is an atom), success of the query $?-not C$ in a certain context (given by $\mathcal{T}(\Pi)$) means that this constraint is fulfilled in the admissible resource-based answer sets Π selected by that context. If the context where $?-not C$ is executed results from a query $?-A$, this implies by Theorem 3.2 that $\leftarrow C$ is fulfilled at least one admissible resource-based answer set including A . So, in admissible programs one should identify and check (a posteriori) constraints that are *relevant* to the query according to Definition 2.4.

4 Concluding Remarks

A relevant question about RAS-XSB-resolution is whether it might be applicable to non-ground queries and programs. By resorting to standard unification, non-ground queries on ground programs can be easily managed. In future work we intend however to extend the procedure to non-ground programs without requiring preliminary program grounding. This should be made possible by the tabling mechanism, which stores ground positive and negative intermediate results, and by Relevance and Modularity of resource-based answer set semantics.

An important issue is whether RAS-XSB-resolution might be extended to plain ASP. Unfortunately, ASP programs may have a quite complicated structure: the effort of (Gebser et al. 2009) has been, in fact, that of performing a layer-based computation upon some conditions. Many answer set programs concerning real applications are however already expressed with constraints at the top layer, as required by our approach.

A comparison with existing proof procedures can be only partial, as these procedures cope with any answer set program, with its involved internal structure. So, overall our procedure imposes less 'a priori' conditions and has a simple definition, but this is obtained by means of a strong preliminary assumption about constraints. However, as the expressive power and complexity remain the same, our approach might constitute a way of simplifying implementation aspects without significant losses in "practical" expressivity.

We intend to investigate an integration of RAS-XSB-resolution with principles and techniques introduced in (Bonatti et al. 2008), so as to further enlarge its applicability to what they call *finitary programs*, which are a large class of non-ground programs with function symbols. In fact, this approach allows programmers to make use of popular recursive definitions which are common in Prolog, and makes ASP technology even more competitive with respect to other state-of-the-art techniques.

In summary, we have proposed the theoretical foundations of a proof procedure related to a reasonable extension of answer set programming. The procedure has been obtained by taking as a basis XSB-resolution and its tabling features. Future work includes a precise design of a RAS-XSB-resolution implementation. Our objective is to realize an efficient inference engine, that should then be checked and experimented on (suitable versions of) well-established benchmarks (see, e.g., (Calimeri et al. 2016)). We intend in this sense to seek an integration with XSB, and with well-established ASP-related systems (cf. the discussion in (Giunchiglia et al. 2008)), already used for the implementation of the procedure proposed in (Bonatti et al. 2008).

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This appendix contains background material concerning ASP (App. A), Resource-based ASP (App. B), and XSB-resolution (App. C). (All notions have been borrowed from the cited literature). Appendix D contains the proofs of the results in Section 3.1.

A Background on ASP

We refer to the standard definitions concerning propositional general logic programs, as reported, for instance, in (Apt and Bol 1994; Lloyd 1993; Gelfond and Lifschitz 1988). We will sometimes re-elaborate definitions and terminology (without substantial change), in a way which is functional to the discussion.

In the answer set semantics (originally named “stable model semantics”), an answer set program Π (or simply “program”) is a finite collection of *rules* of the form $H \leftarrow L_1, \dots, L_n$, where H is an atom, $n \geq 0$ and each literal L_i is either an atom A_i or its *default negation* $\text{not}A_i$. The left-hand side and the right-hand side of rules are called *head* and *body*, respectively. A rule can be rephrased as $H \leftarrow A_1, \dots, A_m, \text{not}A_{m+1}, \dots, \text{not}A_n$, where A_1, \dots, A_m can be called *positive body* and $\text{not}A_{m+1}, \dots, \text{not}A_n$ can be called *negative body*.² A rule with empty body ($n = 0$) is called a *unit rule*, or *fact*. A rule with empty head, of the form $\leftarrow L_1, \dots, L_n$, is a *constraint*, and it states that the literals L_1, \dots, L_n cannot be simultaneously true. A positive program is a logic program including no negative literals and no constraints.

For every atom A occurring in a rule of program Π either as positive literal A or in a negative literal $\text{not}A$, we say that A occurs in Π . Therefore, as Π is by definition finite it is possible to determine the set S_Π composed of all the atoms occurring in Π .

In the rest of the paper, whenever it is clear from the context, by “a (logic) program Π ” we mean an answer set program (ASP program) Π . As it is customary in the ASP literature, we will implicitly refer to the *ground* version of Π , which is obtained by replacing in all possible ways the variables occurring in Π with the constants occurring in Π itself, and is thus composed of ground atoms, i.e., atoms which contain no variables. We do not consider “classical negation” (cf., (Gelfond and Lifschitz 1991)), nor we consider double negation $\text{notnot}A$. We do not refer (at the moment) to the various useful programming constructs defined and added over time to the basic ASP paradigm.

A program may have several answer sets, or may have no answer set (while in many semantics for logic programming a program admits exactly one “model”, however defined). Whenever a program has no answer sets, we will say that the program is *inconsistent*. Correspondingly, checking for consistency means checking for the existence of answer sets.

Consistency of answer set programs is related, as it is well-known, to the occurrence of *negative cycles*, (or negative “loops”) i.e. cycles through negation, and to their connections to other parts of the program (cf., e.g., (Costantini 2006)).

To clarify this matter, some preliminary definitions are in order.

Definition A.1 (Dependency Graph)

For a ground logic program Π , the dependency graph G_Π is a finite directed graph whose vertices are the atoms occurring in Π (both in positive and negative literals). There is a positive (resp. negative) edge from vertex R to vertex R' iff there is a rule ρ in Π with R as its head where

² Observe that an answer set program can be seen as a Datalog program with negation —cf., (Lloyd 1993; Apt and Bol 1994) for definitions about logic programming and Datalog.

R' occurs positively (resp. negatively) in its body, i.e. there is a positive edge if R' occurs as a positive literal in the body of ρ , and a negative edge if R' occurs in a negative literal $not R'$ in the body of ρ . We say that:

- R depends on R' if there is a path in G_Π from R to R' ;
- R depends positively on R' if there is a path in G_Π from R to R' containing only positive edges;
- R depends negatively on R' if there is a path in G_Π from R to R' containing at least one negative edge.
- there is an acyclic dependency of R on R' if there is an acyclic path in G_Π from R to R' ; such a dependency is even if the path comprises an even number of edges, is odd otherwise.

In this context we assume that R depends on itself only if there exist a non-empty path in G_Π from R to itself. (Note that empty paths are excluded, otherwise each R would always depend -positively- upon itself by definition).

By saying that atom A depends (positively or negatively) upon atom B , we implicitly refer to the above definition.

Definition A.2 (Cycles)

A cycle in program Π corresponds to a circuit occurring in G_Π . We say that:

- a positive cycle is a cycle including only positive edges;
- a negative cycle is a cycle including at least one negative edge;
- given a negative cycle C , we say that C is odd (or that C is an odd cycle) if C includes an odd number of negative edges;
- given a positive cycle C , we say that C is even (or that C is an even cycle) if C includes an even number of negative edges;

When referring to positive/negative even/odd cycles we implicitly refer to the above definition.

Below is the formal specification of the answer set semantics, elaborated from (Gelfond and Lifschitz 1988). Preliminarily, we remind the reader that the least Herbrand model of a positive logic program Π can be computed by means of its immediate consequence operator T_Π , that can be defined as follows (the original definition is due to Van Emden and Kowalski). We then introduce the definition of reduct, the Γ operator and finally the definition of answer set. Given a positive program Π and a set of atoms I , let

$$T_\Pi(I) = \{A : \text{there exists a rule } A \leftarrow A_1, \dots, A_m \text{ in } \Pi \text{ where } \{A_1, \dots, A_m\} \subseteq I\}$$

The T_Π operator always has a unique least fixpoint, that for finite propositional programs is computable in a finite number of steps.

The following definition of (GL-)reduct is due to Gelfond and Lifschitz.

Definition A.3

Let I be a set of atoms and Π a program. The reduct of Π modulo I is a new program, denoted as Π^I , obtained from Π by: 1. removing from Π all rules which contain a negative literal $not A$ such that $A \in I$; and by 2. removing all negative literals from the remaining rules.

Notice that for each negative literal $not A$ which is removed at step 2, it holds that $A \notin I$: otherwise, the rule where it occurs would have been removed at step 1. We can see that Π^I is a positive logic program. Answer sets are defined as follows, via the GL-operator Γ .

Definition A.4 (The GL-Operator Γ)

Let I be a set of atoms and Π a program. We denote with $\Gamma_\Pi(I)$ the least Herbrand model of Π^I .

Definition A.5

Let I be a set of atoms and Π a program. I is an *answer set* of Π if and only if $\Gamma_\Pi(I) = I$.

Answer sets form an anti-chain with respect to set inclusion. The answer set semantics extends the well-founded semantics (wfs), formally introduced in (Van Gelder et al. 1991) and then further discussed and characterized (cf. (Apt and Bol 1994) for a survey), that provides a unique three-valued model. The well-founded model $wfs_\Pi = \langle W^+, W^- \rangle$ of program Π is specified by making explicit the set of true and false atoms, all the other atoms implicitly assuming the truth value “undefined”. Intuitively, according to the wfs:

- The set W^+ is the set of atoms which can be derived top-down, say, like in Prolog, without incurring in cycles.
- The set W^- is the set of atoms which cannot be derived either because they are not the head of any rule, or because every possible derivation incurs in a positive cycle, or because every possible derivation incurs in some atom which in turn cannot be derived.
- The undefined atoms are those atoms which cannot be derived because every possible derivation incurs in a negative cycle.

Some of the classical models of Π (interpreted in the obvious way as a classical first-order theory, i.e. where the comma stands for conjunction and the symbol \leftarrow stands for implication) can be answer sets, according to some conditions introduced in what follows.

Definition A.6

Given a non-empty set of atoms I and a rule ρ of the form $A \leftarrow A_1, \dots, A_n, \text{not} B_1, \dots, \text{not} B_m$, we say that ρ is *supported* in I iff $\{A_1, \dots, A_n\} \subseteq I$ and $\{B_1, \dots, B_m\} \cap I = \emptyset$.

Definition A.7

Given a program Π and a non-empty set of atoms I , we say that I is *supported* w.r.t. Π (or for short Π -supported) iff $\forall A \in I$, A is the head of a rule ρ in Π which is supported in I .

Answer sets of Π , if any exists, are supported minimal classical models of the program. They however enjoy a stricter property, that we introduce below (cf., Proposition A.2).

Definition A.8

Given a program Π and a set of atoms I , an atom $A \in I$ is *consistently supported* w.r.t. Π and I iff there exists a set S of rules of Π such that the following conditions hold (where we say that A is consistently supported via S):

1. every rule in S is supported in I ;
2. exactly one rule in S has conclusion A ;
3. A does not occur in the positive body of any rule in S ;
4. every atom B occurring in the positive body of some rule in S is in turn consistently supported w.r.t. Π and I via a set of rules $S' \subseteq S$.

Note that A cannot occur in the negative body of any rule in S either, since all such rules are supported in I . S is called a *consistent support set* for A (w.r.t. Π and I). Moreover, by condition (ii), different support sets for A may exist, each one including a different rule with head A .

Definition A.9

Given a program Π and a set of atoms I , we say that I is a *consistently supported* set of atoms (w.r.t. Π) iff $\forall A \in I$, A is consistently supported w.r.t. Π and I . We say that I is a *maximal consistently supported* set of atoms (MCS, for short) iff there does not exist $I' \supset I$ such that I' is consistently supported w.r.t. Π . We say, for short, that I is an MCS for Π .

Observe that an MCS can be empty only if it is unique, i.e., only if no non-empty consistently supported set of atoms exists. In both the answer set and the well-founded semantics atoms involved/defined exclusively in positive cycles are assigned truth value *false*. However, the answer set semantics tries to assign a truth value to atoms involved in negative cycles, which are *undefined* under the well-founded semantics (precisely, it succeeds in doing so if the given program Π is consistent). Therefore, for every answer set M , $W^+ \subseteq M$. It is easy to see that:

Proposition A.1

Given the well-founded model $\langle W^+, W^- \rangle$ of program Π , W^+ is a consistently supported set of atoms.

Notice that W^+ is not in general an MCS, as the following proposition holds:

Proposition A.2

Any answer set M of program Π is an MCS for Π .

However, maximal consistently supported sets of atoms are not necessarily answer sets.

We introduce some useful properties of answer set semantics from (Dix 1995).

Definition A.10

The sets of atoms a single atom A depends upon, directly or indirectly, positively or negatively, is defined as $dependencies_of(A) = \{B : A \text{ depends on } B\}$.

Definition A.11

Given a program Π and an atom A , $rel_rul(\Pi; A)$ is the set of relevant rules of Π with respect to A , i.e. the set of rules that contain an atom $B \in (\{A\} \cup dependencies_of(A))$ in their heads.

The notions introduced by Definitions A.10 and A.11 for an atom A can be plainly generalized to sets of atoms. Notice that, given an atom (or a set of atoms) X , $rel_rul(\Pi; X)$ is a subprogram of Π .

An ASP program can be seen as divided into components, some of them involving cyclic dependencies.

Definition A.12

An answer set program Π is *cyclic* if for every atom A occurring in the head of some rule ρ in Π , it holds that $A \in dependencies_of(A)$. In particular, Π is *negatively* (resp., *positively*) *cyclic* if some (resp., none) of these dependencies is negative. A program Π in which there is no head A such that $A \in dependencies_of(A)$ is called *acyclic*.

A cyclic program is not simply a program including some cycle: rather, it is a program where every atom is involved in some cycle. It is easy to see the following.

- An acyclic program has a unique (possibly empty) answer set, coinciding with the set W^+ of true atoms of its well-founded model. Acyclic programs coincide with *stratified* programs in a well-known terminology (Apt and Bol 1994). We prefer to call them 'acyclic' as the notion of strata is irrelevant in the present context.

- A positively cyclic program has a unique empty answer set, coinciding with the set W^+ of true atoms of its well-founded model.
- Negatively cyclic programs have no answer sets and have an empty well-founded model, in the sense that all atoms occurring in such a program are undefined under the well-founded semantics.

In the following, unless explicitly specified by a “cyclic program” (or program component) we intend a negatively cyclic program (or program component, i.e. a subprogram of a larger program). By Definition A.12, there exist programs that are neither cyclic nor acyclic, though involving cyclic and/or acyclic fragments as subprograms, where such fragments can be either independent of or related to each other.

Definition A.13

A subprogram Π_s of a given program Π is *self-contained* (w.r.t. Π) if the set X of atoms occurring (either positively or negatively) in Π_s is such that $rel_rul(\Pi; X) \subseteq \Pi_s$.

Notice that a subprogram $\Pi_s = \Pi$ is self-contained by definition.

Definition A.14

Given two subprograms Π_{s_1}, Π_{s_2} of a program Π , Π_{s_2} is *on top* of Π_{s_1} if the set X_2 of atoms occurring in the head of some rule in Π_{s_2} is such that $rel_rul(\Pi; X_2) \subseteq \Pi_{s_2} \cup \Pi_{s_1}$, and the set X_1 of atoms occurring (either positively or negatively) only in the body of rules of Π_{s_2} is such that $rel_rul(\Pi; X_1) \subseteq \Pi_{s_1}$.³

Notice that, by Definition A.14, if Π_{s_2} is *on top* of Π_{s_1} , then X_1 is a *splitting set* for Π in the sense of (Lifschitz and Turner 1994).

Definition A.15

A program obtained as the union of a set of cyclic or acyclic programs, none of which is on top of another one, is called a *jigsaw* program.

Thus any program/component, either acyclic or cyclic or jigsaw, can possibly but not necessarily be self-contained. An entire program is self-contained, but not necessarily jigsaw. We introduce a useful terminology for jigsaw programs which are self-contained.

Definition A.16

Let Π be a program and Π_s a jigsaw subprogram of Π . Then, Π_s is *standalone* (w.r.t. Π) if it is self-contained (w.r.t. Π).

In case we refer to a standalone program Π_s without mentioning the including program Π , we intend Π to be identifiable from the context.

The following property states that a program can be divided into subprograms where a standalone one can be understood as the bottom *layer*, which is at the basis of a “tower” where each level is a jigsaw subprogram standing on top of lower levels.

³ This notion was introduced in (Costantini 1995; Lifschitz and Turner 1994).

Proposition A.3

A non-empty answer set program Π can be seen as divided into a sequence of *components*, or layers, C_1, \dots, C_n , $n \geq 1$ where: C_1 , which is called the *bottom* of Π , is a standalone program; each component C_i , for $i > 1$, is a jigsaw program which is on top of $C_{i-1} \cup \dots \cup C_1$.

In fact, the bottom layer (that may coincide with the entire program) necessarily exists as the program is finite, and so does any upper layer. The advantage of such a decomposition is that, by the *Splitting Theorem* introduced in (Lifschitz and Turner 1994), the computation of answer sets of Π can be divided into subsequent phases.

Proposition A.4

Consider a non-empty ASP program Π , divided according to Proposition A.3 into components C_1, \dots, C_n , $n \geq 1$. An answer set S of Π (if any exists) can be computed incrementally as follows:

- step 0. Set $i = 1$.
- step 1. Compute an answer set S_i of component C_i (for $i = 1$, this accounts to computing an answer set of the standalone bottom component).
- step 2. Simplify program C_{i+1} by: (i) deleting all rules in which have *not* B in their body, for some $B \in S_i$; (ii) deleting (from the body of the remaining rules) every literal *not* F where F does not occur in the head of rules of C_{i+1} and $F \notin S_i$, and every atom E with $E \in S_i$.⁴
- step 3. If $i < n$ set $i = i + 1$ and go to step 1, else set $S = S_1 \cup \dots \cup S_n$.

All answer sets of Π can be generated via backtracking (from any possible answer set of C_1 , combined with any possible answer set of simplified C_2 , etc.). If no (other) answer set of Π exists, then at some stage step 1 will fail. An incremental computation of answer sets has also been adopted in (Gebser et al. 2009).

B Background on Resource-Based Answer Set Semantics

The following formulation of resource-based answer set semantics is obtained by introducing some modifications to the original definition of the answer set semantics. Some preliminary elaboration is needed. Following Proposition A.3, a nonempty answer set program Π (that below we call simply “program”) can be seen as divided into a sequence of *components*, and, based upon such a decomposition, as stated in Proposition A.4, the answer sets of a program can be computed incrementally in a bottom-up fashion. Resource-based answer sets can be computed in a similar way. Therefore, we start by defining the notion of resource-based answer sets of standalone programs.

The semantic variation that we propose implies slight modifications in the definition of the T_Π and the Γ operator, aimed at forbidding the derivation of atoms that necessarily depend upon their own negation. The modified reduct, in particular, keeps track of negative literals which the “traditional” reduct would remove.

Definition B.1

Let I be a set of atoms and let Π be a program. The *modified reduct* of Π modulo I is a new program, denoted as $\hat{\Pi}^I$, obtained from Π by removing from Π all rules which contain a negative premise *not* A such that $A \in I$.

⁴ Notice that, due to the simplification, C_{i+1} becomes standalone.

For simplicity, let us consider each rule of a program as reordered by grouping its positive and its negative literals, as follows:

$$A \leftarrow A_1, \dots, A_m, \text{not} B_1, \dots, \text{not} B_n$$

Moreover, let us define a *guarded atom* to be any expression of the form $A||G$ where A is an atom and $G = \{\text{not} C_1, \dots, \text{not} C_\ell\}$ is a possibly empty collection of $\ell \geq 0$ negative literals. We say that A is guarded by the C_i s, or that G is a guard for A .

We define a modified T_Π which derives only those facts that do not depend (neither directly nor indirectly) on their own negation. The modified T_Π operates on sets of guarded atoms. For each inferred guarded atom $A||G$, the set G records the negative literals A depends on.

Definition B.2 (Modified T_Π)

Given a propositional program Π , let

$$\begin{aligned} T_\Pi(I) = & \left\{ A||G_1 \cup \dots \cup G_r \cup \{\text{not} B_1, \dots, \text{not} B_n\} : \text{there exists a rule} \right. \\ & A \leftarrow A_1, \dots, A_r, \text{not} B_1, \dots, \text{not} B_n \text{ in } \Pi \text{ such that} \\ & \left. \{A_1||G_1, \dots, A_r||G_r\} \subseteq I \text{ and } \text{not} A \notin \{\text{not} B_1, \dots, \text{not} B_n\} \cup G_1 \cup \dots \cup G_r \right\}. \end{aligned}$$

For each $n \geq 0$, let T_Π^n be the set of guarded atoms defined as follows:

$$\begin{aligned} T_\Pi^0 &= \{A||\emptyset : \text{there exists unit rule } A \leftarrow \text{ in } \Pi\} \\ T_\Pi^{n+1} &= T_\Pi(T_\Pi^n) \end{aligned}$$

The *least contradiction-free Herbrand set* of Π is the following set of atoms:

$$\hat{T}_\Pi = \{A : A||G \in T_\Pi^i \text{ for some } i \geq 0\}.$$

Notice that the least contradiction-free Herbrand set of a (modified reduct of a) program, does not necessarily coincide with the full least Herbrand model of the “traditional” reduct, as its construction excludes from the result those atoms that are guarded by their own negation. We can finally define a modified version of the Γ operator.

Definition B.3 (Operator $\hat{\Gamma}$)

Let I be a set of atoms and Π a program. Let $\hat{\Pi}^I$ be the modified reduct of Π modulo I , and J be its least contradiction-free Herbrand set. We define $\hat{\Gamma}_\Pi(I) = J$.

It is easy to see that given a program Π and two sets I_1, I_2 of atoms, if $I_1 \subseteq I_2$ then $\hat{\Gamma}_\Pi(I_1) \supseteq \hat{\Gamma}_\Pi(I_2)$. Indeed, the larger I_2 leads to a potentially smaller modified reduct, since it may causes the removal of more rules.

For technical reasons, we need to consider potentially supported sets of atoms.

Definition B.4

Let Π be a program, and let I be a set of atoms. I is Π -based iff for any $A \in I$ there exists rule ρ in Π with head A .

It can be shown (see, (Costantini and Formisano 2015)) that, given a standalone program Π and a non-empty Π -based set I of atoms, and given $M = \hat{\Gamma}_\Pi(I)$, if $M \subseteq I$ then M is a consistently supported set of atoms for Π . Consequently, we have that M is an MCS (cf., Definition A.9) for Π iff there exists I such that $M \subseteq I$, and there is no proper subset I_1 of I such that $\hat{\Gamma}_\Pi(I_1) \subseteq I_1$. We now define resource-based answer sets of a standalone program.

Definition B.5

Let Π be a standalone program, and let I be a Π -based set of atoms. $M = \hat{\Gamma}_{\Pi}(I)$ is a *resource-based answer set* of Π iff M is an MCS for Π .

It is easy to see that any answer set of a standalone program Π is a resource-based answer set of Π and, if Π is acyclic, the unique answer set of Π is the unique resource-based answer set of Π . These are consequences of the fact that consistent ASP programs are non-contradictory, and the modified T_{Π} , in absence of contradictions (i.e. in absence of atoms necessarily depending upon their own negations), operates exactly like T_{Π} . In case of acyclic programs, the unique answer set I is also the unique MCS as the computation of the modified reduct does not cancel any rule, and the modified T_{Π} can thus draw the maximum set of conclusions, coinciding with I itself.

Being an MCS, a resource-based answer set can be empty only if it is the unique resource-based answer set.

Below we provide the definition of resource-based answer sets of a generic program Π .

Definition B.6

Consider a non-empty ASP program Π , divided according to Proposition A.3 into components C_1, \dots, C_n , $n \geq 1$. A resource-based answer set S of Π is defined as $M_1 \cup \dots \cup M_n$ where M_1 is a resource-based answer set of C_1 , and each M_i , $1 < i \leq n$, is a resource-based answer set of standalone component C'_i , obtained by simplifying C_i w.r.t. $M_1 \cup \dots \cup M_{i-1}$, where the simplification consists in: (i) deleting all rules in C_i which have *not* B in their body, $B \in M_1 \cup \dots \cup M_{i-1}$; (ii) deleting (from the body of remaining rules) every literal *not* D where D does not occur in the head of rules of C_i and $D \notin M_1 \cup \dots \cup M_{i-1}$, and also every atom D with $D \in M_1 \cup \dots \cup M_{i-1}$.⁵

Definition B.6 brings evident analogies to the procedure for answer set computation specified in Proposition A.4. This program decomposition is under some aspects reminiscent of the one adopted in (Gebser et al. 2009). However, in general, resource-based answer sets are not models in the classical sense: rather, they are Π -supported sets of atoms which are the wider subsets of some classical model that fulfills non-contradictory support. We can prove, in fact, the following result:

Theorem B.1

A set of atoms I is a resource-based answer set of Π iff it is an MCS for Π .

Resource-based answer sets still form (like answer sets) an anti-chain w.r.t. set inclusion, and answer sets (if any) are among the resource-based answer sets. Clearly, resource-based answer sets semantics still extends the well-founded semantics. Differently from answer sets, a (possibly empty) resource-based answer set always exists.

It can be observed that complexity remains the same as for ASP. In fact:

Proposition B.1

Given a program Π , the problem of deciding whether there exists a set of atoms I which is a resource-based answer set of Π is NP-complete.

⁵ Notice that, due to the simplification, C'_i is standalone.

C XSB-resolution in a Nutshell

Below we briefly illustrate the basic notions of XSB-resolution. An ample literature exists for XSB-resolution, from the seminal work in (Chen and Warren 1993) to the most recent work in (Swift and Warren 2012) where many useful references can also be found. XSB resolution is fully implemented, and information and downloads can be found on the XSB web site, xsb.sourceforge.net/index.html.

XSB-resolution adopts *tabling*, that will be useful for our new procedure. Tabled logic programming was first formalized in the early 1980's, and several formalisms and systems have been based both on tabled resolution and on magic sets, which can also be seen as a form of tabled logic programming (c.f. (Swift and Warren 2012) for references). In the Datalog context, tabling simply means that whenever atom S is established to be true or false, it is recorded in a table. Thus, when subsequent calls are made to S , the evaluation ensures that the answer to S refers to the record rather than being re-derived using program rules. Seen abstractly, the table represents the given state of a computation: in this case, subgoals called and their answers so far derived. One powerful feature of tabling is its ability to maintain other global elements of a computation in the “table”, such as information about whether one subgoal depends on another, and whether the dependency is through negation. By maintaining this global information, tabling is useful for evaluating logic programs under the well-founded semantics. Tabling allows Datalog programs with negation to terminate with polynomial data complexity under the well-founded semantics.

An abridged specification of the basic concepts underlying XSB-resolution is provided below for the reader's convenience. We refer the reader to the references for a proper understanding. We provide explanations tailored to ground (answer set) programs, where a number of issues are much simpler than the general case (non-ground programs and, particularly, programs with function symbols). For definitions about procedural semantics of logic programs we again refer to (Lloyd 1993; Apt and Bol 1994), and in particular we assume that the reader is to some extent acquainted with the SLD-resolution (Linear resolution with Selection function for Definite programs) and SLDNF-resolution (for logic programs with Negation-as-Failure) proof procedures, which form the computational basis for Prolog systems. Briefly, a ground negative literal succeeds under SLDNF-resolution if its positive counterpart finitely fails, and vice versa it fails if its positive counterpart succeeds. SLDNF-resolution has the advantage of goal-oriented computation and has provided an effective computational basis for logic programming, but it cannot be used as inference procedure for programs including either positive or negative cycles.

XSB-resolution stems from SLS-resolution (Przymusiński 1989; Ross 1992), which is correct and complete w.r.t. the well-founded semantics, via the ability to detect both positive cycles, which make involved atoms false w.r.t. the wfs, and negative cycles, which make the involved atoms undefined. Later, solutions with “memoing” (or “tabling”) have been investigated, among which (for positive programs) OLD-resolution (Tamaki and Sato 1986), which maintains a table of calls and their corresponding answers: thus, later occurrences of the same calls can be resolved using answers instead of program rules. An effective variant of SLS with memoing and simple methods for loop detection is XOLD-resolution (Chen and Warren 1993), which builds upon OLD-resolution. SLG-resolution (Chen and Warren 1996) is a refinement of XOLD-resolution, and is actually the basis of implemented XSB-resolution. In SLG, many software engineering aspects and implementation issues are taken into account. In this context, as we still do not treat practical

implementation issues it is sufficient to introduce basic concepts related to SLS and XOLDTNF-resolution.

As done before, let us consider each rule of a program as reordered by grouping its positive and its negative literals, as follows: $A \leftarrow A_1, \dots, A_m, \text{not} B_1, \dots, \text{not} B_n$. Moreover, let be given a *goal* of the form $\leftarrow L_1, \dots, L_k$, where the L_i s are literals, let us consider a *positivistic computation rule*, which is a computation rule that selects all positive literals before any negative ones. These assumptions were originally required by SLS and have been dropped later, but they are useful to simplify the illustration.

The basic building block of SLS-resolution is the *SLP-tree*, which deals with goals of the form $\leftarrow Q$, that form the root of the tree. For each positive subgoal which is encountered, its SLP sub-tree is built basically as done in SLD-resolution. Leaves of the tree can be:

- *dead leaves*, i.e. nodes with no children because either there is no program rule to apply to the selected atom A , or because A was already selected in an ancestor node (situation which correspond to a positive cycle); in both case the node is *failed*;
- *active leaves*, which are either empty (*successful node*) or contain only negative subgoals.

More precisely, the *Global tree* T for goal $\leftarrow Q$ is built as follows.

- Its root node is the SLP-tree for the original goal.
- Internal tree nodes are SLP-trees for intermediate positive sub-goals.
- *Negation nodes* are created in correspondence of negative subgoals occurring in non-empty active leaves.

The management of negation node works as follows: the negation node corresponding to sub-goal $\text{not}A$ is developed into the SLP-tree for A , unless in case such a node already exists in the tree (negative cycles detection). Then: if some child of a negation node J is a successful tree node, then J is *failed*; if every child of a negation node J is either a failed node or a dead leaf, then J is *successful*.

Any node that can be proved successful or failed is *well-determined*, and any node which is not well-determined is *undetermined*. A *successful branch* of T is a branch that ends at a successful leaf and corresponds to success of the original goal. A goal which leads via any branch to an undetermined node is undetermined. Otherwise, the goal is failed.

It has been proved that a successful goal is composed of literals which are true w.r.t. the wfs, a failed goal includes some literal which is false w.r.t. the wfs, and an undetermined goal includes some literal which is undefined.

XOLDTNF-resolution augments SLS-resolution with tabling and with a simple direct way for negative cycles detection. In the following, given a program Π , let $\mathcal{T}(\Pi)$ be the data structure used by the proof procedure for tabling purposes, i.e. the table associated with the program (or simply “program table”). The improvements of XOLDTNF over SLS are mainly the following.

- The Global tree is split into several trees, one for each call, whose root is an atom A . As soon as the call leads to a result, the “answer”, i.e. the truth value of A , is recorded in the table. Only *true* or *undefined* answers are explicitly recorded. Whenever A should occur in a non-root node, it can be resolved only by the answer that has been computed and recorded in $\mathcal{T}(\Pi)$ or that can be computed later. This avoids positive loops. An atom whose associated tree has in the end no answer leaf has truth value *false* because either no applicable program rule exists, or a positive cycle has been encountered.

- For detecting negative cycles the method introduced in Definition 3.1 is adopted.

For Datalog programs, XOLDTNF-resolution is, like SLS-resolution, correct and complete w.r.t. the wfs. Consequently, so are SLG- and XSB-resolution.

D Proofs from Section 3.1

This section contains the proofs of Theorems 3.1 and 3.2 and some preliminary results.

Lemma D.1

Let Π be an acyclic program. RAS-XSB-resolution is correct and complete w.r.t. such a program.

Proof

An acyclic program is stratified and thus admits a two-valued well-founded model (i.e. no atom is undefined) where W^+ coincides with the unique (resource-based) answer set. XSB-resolution is correct and complete w.r.t. such a program. Thus, any literal occurring in Π either definitely succeeds by case 1 of RAS-XSB-resolution or definitely fails by case 2.b of RAS-XSB resolution (cf., Definition 3.5). Since such cases just resort to plain XSB-resolution, this concludes the proof. \square

Lemma D.2

Let Π be a cyclic program. RAS-XSB-resolution is correct and complete w.r.t. such a program.

Proof

Let M be a resource-based answer set of Π . We prove that, for every $A \in M$, query $?-A$ succeeds under RAS-XSB-resolution. M (which is a maximal consistently supported set of atoms (MCS)) can be obtained by applying the modified immediate consequence operator) to some Π -based set of atoms I . From the application of the modified $T_{\Pi'}$ we can trace back a set of program rules from which A can be proved via RAS-XSB-resolution (cases 1 and 3 of Definition 3.5). Notice first that $T_{\Pi'}^0 = \emptyset$ as a cyclic program includes no fact. (Recall that, by definition, a program is cyclic if each of its heads depends directly or indirectly on itself.) However, Π' necessarily contains some rule with body including negative literals only, thus leading to a nonempty $T_{\Pi'}^1$ and determining a final non-empty result of repeated application of $T_{\Pi'}$. For some $i \geq 1$ there will be $A || G \in T_{\Pi'}^i$ (for a guard G). This means that there exists a rule ρ in Π' which is applicable, i.e. A does not occur in its body, and $notA$ does not occur in the guard. Let $B_1, \dots, B_n, notC_1, \dots, notC_m, n, m \geq 0$ be the body of ρ . Since M is an MCS ρ will be supported in M , i.e. it will hold that $B_i \in M, i \leq n$ and $C_j \notin M, j \leq m$.

Let us consider the $notC_j$ s. It cannot be $C_j \in I$, otherwise, by definition of the modified reduct, rule ρ would have been canceled. Moreover, the C_i s are not derived by the modified $T_{\Pi'}$ so allowing for the derivation of A . Being the program cyclic, one of the following must be the case for this to happen.

- C_j is not derived by the modified $T_{\Pi'}$ (which differs from the standard one only concerning guarded atoms) because it depends positively upon itself and so it is false in every resource-based answer set and in the well-founded semantics. In this case $notC_j$ succeeds by case 3.b of RAS-XSB-resolution: in fact C_j fails by case 2.b since XSB-resolution is correct and complete w.r.t. the well-founded semantics.

- C_j is not derived by the modified $T_{\Pi'}$ because it depends negatively upon itself and at some point the derivation incurs in a guard including $\text{not}C_j$. In this case, $\text{not}C_j$ succeeds either by case 3.c or by case 3.d of RAS-XSB-resolution.

For each of the B_i s we can iterate the same reasoning as for A . As noted before, being the program cyclic there are no unit rules, but for M to be nonempty there will exist some rule in Π without positive conditions which is supported in M . Therefore, a RAS-XSB-derivation is always finite. This concludes this part of the proof.

Let us now assume that $?-A$ succeeds by RAS-XSB-resolution. We prove that there exists resource-based answer set M such that $A \in M$. We have to recall that a resource-based answer set M is obtained as $M = \hat{\Gamma}_{\Pi}(I)$ where $M \subseteq I$ for some set of atoms I , and that M is an MCS for Π . Let us refer to Definition 3.5. Since the program is cyclic, then A succeeds via case 1.b, i.e. there exists a rule ρ in Π (where A does not occur in the body), of the form $A \leftarrow B_1, \dots, B_n, \text{not}C_1, \dots, \text{not}C_m$ (for $n, m \geq 0$), where all the B_i s and all the $\text{not}C_j$ s succeed via RAS-XSB-resolution. We have to prove that there exists a resource-based answer set M , which is an MCS for Π , where this rule is supported, i.e. it holds that $B_i \in M$ for all $i \leq n$ and $C_j \notin M$ for all $j \leq m$. From the definition of resource-based answer set, M must be obtained from a set of atoms I , where we must assume to select an I such that $A \in I$, $\{B_1, \dots, B_n\} \subseteq I$ and $\{C_1, \dots, C_m\} \cap I = \emptyset$. So, the modified reduct will cancel all rules in Π with $\text{not}A$ in their body, while keeping ρ . Thus, we have now to prove that ρ allows the modified $T_{\Pi'}$ to add A to M . To this extent, we must consider both the negative and the positive conditions of ρ . Considering the negative conditions, for each the $\text{not}C_j$ s we can observe that, being Π cyclic, one of the following must be the case.

- $\text{not}C_j$ succeeds via either case 3.c or 3.d. It can be one of the following.
 - All rules with head C_j have been canceled by the modified reduct, and so the modified $T_{\Pi'}$ cannot derive C_j .
 - There are rules with head C_j which have not been canceled by the modified reduct, and might thus allow the modified $T_{\Pi'}$ to derive C_j . Since however Π is cyclic, the application of such a rule will be prevented by the occurrence of $\text{not}C_j$ in the guard.
- $\text{not}C_j$ succeeds via case 3.b: in this case, being the program cyclic, C_j depends in every possible way positively upon itself. Thus, C_j cannot be derived by the modified $T_{\Pi'}$ which, apart from guards, works similarly to the standard immediate consequence operator.

For each of the B_i s we can iterate the same reasoning as done for A , and this concludes the proof. \square

Lemma D.3

Let Π be a standalone program. RAS-XSB-resolution is correct and complete w.r.t. such a program.

Proof

The result follows from Lemma D.1 and Lemma D.2 as a standalone program is in general a jigsaw program including both cyclic and acyclic components. \square

Proof of Theorem 3.1

As a premise, we remind the reader that, according to Definition B.6, for every resource-based answer set M of Π we have $M = M_1 \cup \dots \cup M_n$, where $C_1 \cup \dots \cup C_n$ are the components of Π and every M_i is a resource-based answer set of the version of C_i obtained via the simplification specified in the same definition. For every $A \in M$, there exists i , $1 \leq i \leq n$, such that $A \in M_i$.

Let M be a resource-based answer set of Π . We prove that, for every $A \in M$, query $?-A$ succeeds under RAS-XSB-resolution. The proof will be by induction.

Induction base. Since C_1 is standalone, then by Lemma D.3 RAS-XSB-resolution is correct and complete w.r.t. M_1 and C_1 .

Induction step. Assume that RAS-XSB-resolution is correct w.r.t. subprogram $C_1 \cup \dots \cup C_i$, $i \leq n$, and its resource-based answer set $M_1 \cup \dots \cup M_i$. We prove that this also holds for subprogram $C_1 \cup \dots \cup C_{i+1}$ and its resource-based answer set $M_1 \cup \dots \cup M_{i+1}$. After the simplification specified in Definition B.6, which accounts to annotating in $\mathcal{T}(\Pi)$ the results of the RAS-XSB derivations of the atoms in M_{i+1} , we have that C_{i+1} becomes standalone, with resource-based answer set M_{i+1} . Then, for $A \in M_{i+1}$ we can perform the same reasoning as for $A \in M_1$, and this concludes the proof. \square

Proof of Theorem 3.2

Given any query $?-A$, the set of rules used in the derivation of A constitutes a subprogram Π_A of Π . Therefore, by correctness and completeness of RAS-XSB-resolution there exists some resource-based answer set M_A of Π_A such that, after the end of the derivation, we have $A \in \mathcal{T}(\Pi) \iff A \in M_A$ and $\text{not}A \in \mathcal{T}(\Pi) \iff A \notin M_A$. By Modularity of resource-based answer set semantics, there exists some resource-based answer set M of Π such that $M_A \subseteq M$ and therefore $A \in M$. So, let us assume that $?-A_1$ succeeds (if in fact it fails, then by correctness and completeness of RAS-XSB-resolution there exist no resource-based answer set of Π including A_1 , and by definition of RAS-XSB-resolution the table is left unchanged). For subsequent query $?-A_2$ one of the following is the case.

- The query succeeds, and the set of rules used in the derivation of A_2 has no intersection with the set of rules used in the derivation of A_1 . Therefore, by Modularity of resource-based answer set semantics we have that $M_{A_1} \cap M_{A_2} = \emptyset$ and there exists resource-based answer set M of Π such that $(M_{A_1} \cup M_{A_2}) \subseteq M$.
- The query succeeds, and the set of rules used in the derivation of A_2 has intersection with the set of rules used in the derivation of A_1 . So, some literal in the proof will succeed by cases 1.a and 3.a of RAS-XSB-resolution, i.e, by table look-up. Therefore, by Modularity of resource-based answer set semantics we have that $M_{A_1} \cap M_{A_2} \neq \emptyset$ and there exists resource-based answer set M of Π such that $(M_{A_1} \cup M_{A_2}) \subseteq M$.
- The query fails, and the set of rules attempted in the derivation of A_2 has no intersection with the set of rules used in the derivation of A_1 . Therefore, we have that simply there not exists resource-based answer set M such that $A_2 \in M$.
- The query fails, and the set of rules used in the derivation of A_2 has intersection with the set of rules used in the derivation of A_1 . So, either some positive literal in the proof will fail by case 1.a of RAS-XSB-resolution or some negative literal in the proof will fail as its positive counterpart succeeds by case 1.a of RAS-XSB-resolution i.e, in both cases, by table look-up. So, success of A_2 is incompatible with the current state of the table, i.e. with success of

A_1 . Therefore, by Modularity of resource-based answer set semantics and by correctness and completeness of RAS-XSB-resolution we have that there not exists resource-based answer set M such that $A_1 \in M$ and $A_2 \in M$ and $M_{A_1} \subseteq M$.

The same reasoning can be iterated for subsequent queries, and this concludes the proof. \square